Artigo

Solução Semi-Analítica por meio da injeção de polímeros

Semi-Analytical solution by polymer's injection

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Resumo

Neste trabalho apresentamos o método semi-analitico para o cálculo da solução de um sistema de leis de conservação, usado na simulação da recuperação de óleo por meio da injeção de polímeros. Por razões econômicas bancos d'água com polímeros são injetados durante o processo de recuperação. Portanto as condições de fronteira são dadas por funções do ´step´. A não linearidade das curvas de fluxo fracionário e as condições de fronteira são responsáveis pelas descontinuidades que surgem na solução do sistema. Dissipações numéricas para discretizações usuais aparecem na aproximação numérica da solução. Usamos composições da solução do problema de Riemann propostas em Johansen (541-566) e o cálculo das curvas de descontinuidades da solução global do sistema de leis de conservação com condições iniciais e de fronteira. Um programa computacional foi desenvolvido para o cálculo de alguns passos necessários, como as interseções das curvas , pontos tangentes, etc. Apresentamos uma aplicação do método analisando a influência da adsorção do polímero pelo meio através do histórico da produção. Este método rápido pode ser usado para pequenos bancos, concentrações de polímero e outros aspectos do processo da injeção de polímeros.

Palavras-chave

problema de Riemann - leis de conservação - recuperação de óleo - semi-analítico

Abstract

In this paper we present a semi-analytical method to compute solution for a conservation laws system used in the simulation by polymer insection in enhanced oil recovery. For economic reasons slugs of polymer-water are injected in the recovery process. Therefore, the boundary conditions are step functions. Non-linearity of practical fractional flow curves and these boundary conditions are responsible for several discontinuities in the solution of the system. Numerical dissipation of usual discretizations smears numerical approximation for the solution. We use compositions of the Riemann problem solutions proposed in [9], and discontinuity curves crossing points to

$$\phi \frac{\partial}{\partial t}(sc + a(c)) + \frac{\partial}{\partial x}(c\,u_w) = 0 \tag{3}$$

As the usual applications of fractional flow theory [13], the Darcy's laws and total velocity are combined in the definition of the water fractional flow function. In the polymer-flooding case, the water viscosity is increased, then the fractional flow is a function of the water saturation and polymer concentration:

$$f(s,c) = \frac{k_{rw}(s)}{k_{rw}(s) + \frac{\mu_w(c)}{\mu_o} k_{ro}(s)} \tag{4}$$

where k_{rw} (k_{ro}) is the water (oil) relative permeability and m_w (m_o) is the water (oil) viscosity.

$$x' = \frac{x}{L} \qquad \text{and} \qquad t' = \frac{(u_o + u_w)\,t}{\phi L}$$

If L denotes the length of the medium, the usual dimensionless variables are:

and we drop the prime in the dimensionless variables.

Therefore, using the fractional flow theory and dimensionless variables we obtain the final equations, which are,

$$\frac{\partial s}{\partial t} + \frac{\partial f(s,c)}{\partial x} = 0 \tag{5}$$

$$\frac{\partial (sc + a(c))}{\partial t} + \frac{\partial (c\, f(s,c))}{\partial x} = 0. \tag{6}$$

Economic feasibility can be estimated using this model and calculation procedures to decide how polymer can be used, selecting the polymer type. This theory includes arbitrary initial and injected conditions, limited solvent solubility in aqueous phase. These effects will lead to insights concerning polymer slug sizing and optimal injected water-polymer ratios.

The description of the polymer adsorption is given by a Langmuir-type isotherm [11], a concave increasing function:

$$a(c) = \frac{a_1(c)}{1 + a_2(c)}. \tag{7}$$

where *c* and *a(c)* are the species concentration in the aqueous and on the rock phases. In this equation, a_2 controls the curvature of the isotherm and the ratio determines the plateau value for adsorption. A common way to report polymer retention is the polymer solution volume and pore volume ratio.

In this paper we will use the Riemann solutions for the system (5)-(6) to design an algorithm for polymer displacements. We could classify this algorithm as a front-tracking method. However, in the smooth regions – outside neighborhoods of the fronts – the continuous solution is computed by the characteristics method. Otherwise, the discontinuity curves are computed, step by step, according to a new Riemann problem. The left and right states of this Riemann problem are defined by the interactions between discontinuity curves themselves or between them and the characteristics transporting different values for water-polymer mixture saturation and polymer concentration.

We stress the difference from the front tracking method used in [5] and [6], where numerical approximations are used to calculate the smooth part of the solution. Using our procedure the computer time will be significantly smaller.

Some ideas of the proposed method were used in graphical procedures in [1],[2] and [14]. Besides the limitations of graphical procedure, they do not use the Riemann problem solution to justify their procedure.

For $t < T$ the solution can be calculated by the Buckley-Leverett theory for oil displacement by the water flood process. The spreading waves are defined by constant values along the characteristics, the self-similar solutions. The shocks velocities are calculated using the Rankine-Hugoniot and entropy conditions. The initial condition gives a region of constant water saturation.

The solution of Riemann problem for polymer flooding system was first presented by Isaacson [8]. He studied the particular case where the adsorption can be neglected, $a(c) = 0$. In this case , depending on the left and right state, there are six possibilities for the solution. Each one of these possibilities is a finite number composition of smooth solution (or spreading waves), discontinuous solutions (or shock waves) and constant states. The Lax entropy inequalities were used to distinguish the physically meaningful solution.

In [9] Johansen and Winther generalize the Isaacson study, solving the adsorption case where $a(c)$ is not a zero function. They derived entropy conditions from travelling waves analysis to formulate the general Riemann solutions in terms of rarefaction and shock waves. With these arguments they show that there are fourteen different solutions, depending on left and right states.

$$\frac{\partial u}{\partial t} + A(u)\frac{\partial u}{\partial x} = 0$$

The nonlinear system of conservation laws, (5)-(6) can be written in the quasi-linear form:

where $u = (s,t)$ is the state vector and $A(s,t)$ is the Jacobian matrix

$$A(s,t) = \begin{bmatrix} \dfrac{\partial f(s,c)}{\partial s} & \dfrac{\partial f(s,c)}{\partial c} \\ 0 & \dfrac{f(s,c)}{s+h(c)} \end{bmatrix}$$

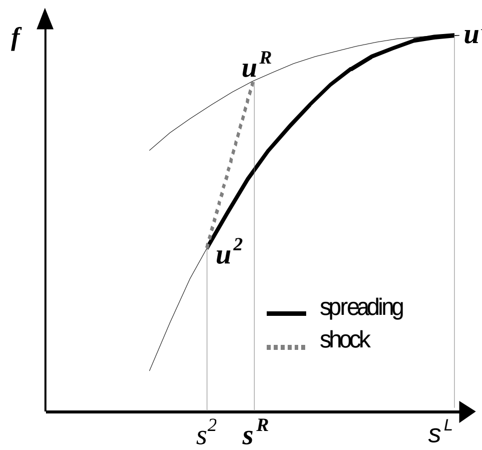

Figure 2

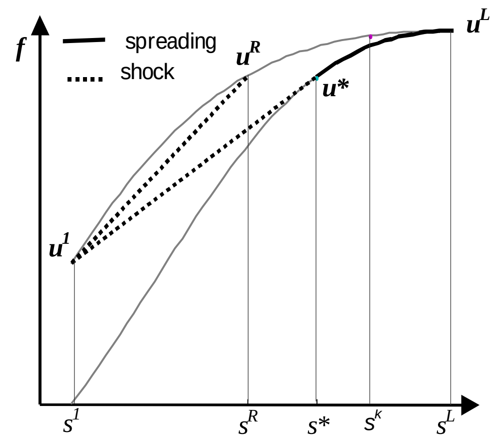

Figure 3

Let $u^* = (s^*, c^L)$ (cf. Figure 1) be the unique saturation, on the polymer-oil fractional curve, such that

$$f_s(s^*, c^L) = \frac{f(s^*, c^L)}{s^* + h_L(c^R)}$$

We see that s^* is at the tangent to the polymer-oil fractional flow curve of a straight line passing through the point $(-h_L(c^R), 0)$, as it is shown in Figure 1.

As shown in Figure 1, the u^*-tangent intersect the water-oil fractional curve, $f(s,0)$, in two points, the states $u^K=(s^K,0)$ and $u^1=(s^1,0)$.The inequality in (9) has a unique solution if and only if $s^R \leq s^K$. Otherwise, if $s^R > s^K$ the composition is also given in Lemma 7.1 [9]:

$$u^L \xrightarrow{s} u^* \xrightarrow{c} u^1 \xrightarrow{s} u^R$$

According to Figure 3, this composition uses a spreading wave along $f(s,c^L)$, a shock, in saturation and concentration, from u^* to u^1 and another Buckley-Leverett solution along $f(s,0)$.

In these constructions use s^R from the spreading wave of the water slug; these are continuously decaying water saturation values. Therefore, the soluion during the polymer flooding is the composition (8), if $s^R \leq s^K$ or composition (10), if $s^R > s^K$.

To obtain the water saturation profile $s(x)$ at a given time we use the time – distance diagram drawn in Figure 4. Streaked areas on this diagram represent spreading waves: the lower area is the Buckley Leverett spreading wave and the upper area are spreading waves from the solution presented in Figure 2.

Taking the water saturation at $x_D = 1$ we can compute the effluent history, the fractional flux at the effluent end, presented in Figure 6. The effluent history provides a means for calculating cumulative oil recovery from the area under the curve:

$$N(t_D) = \int_0^1 (1 - f_w | x_D = 1) dt$$

where $\boldsymbol{N}$ is the cumulative oil recovered expressed as a fraction of the medium pore volume. The cumulative oil recovered are presented in Figure 7.

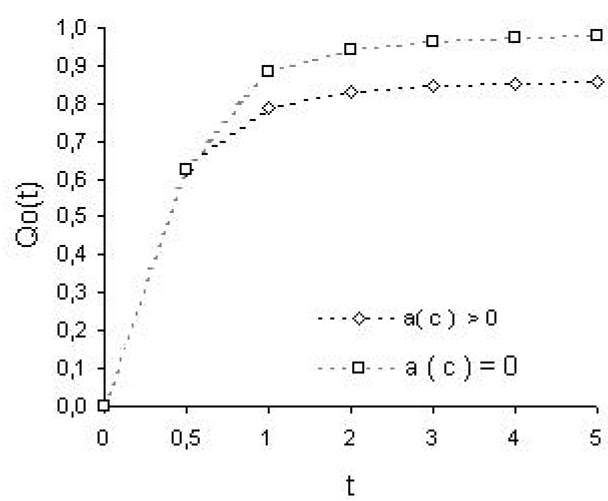

Figure 7

Final Remarks

In this paper we presented a semi-analytical method designed for a oil recovery process in which the water flooding is followed by a continuos polymer-water mixture injection. We also designed a code to compute the features of this recovery process.

We used convenient numerical methods in the following steps:

To find tangent lines, emanating from arbitrary points, to the fractional flow curves; for instance to find s^* in Figure 1.

To find points where straight lines emanating from given points intersect the fractional flow curves, for instance to find s_2 in Figure 2 and s_1 in Figure 1.

To obtain the discontinuity curves plotted in Figure 4. Here a discretization in the spreading waves are used to compute the piecewise approximation for the discontinuity curves. Except for these approximations the method gives the analytical solution.